

# What's to Come is Still Unsure<sup>\*</sup>

## Synthesizing Controllers Resilient to Delayed Interaction

Mingshuai Chen

—Joint work with Martin Fränzle, Yangjia Li, Peter N. Mosaad, Naijun Zhan—

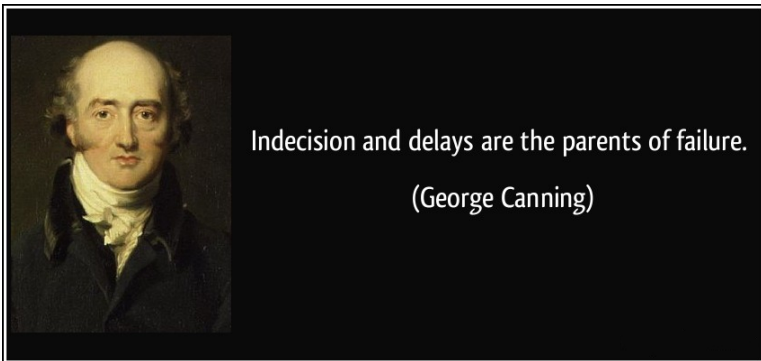


 MISSION · INVAP · February 2022

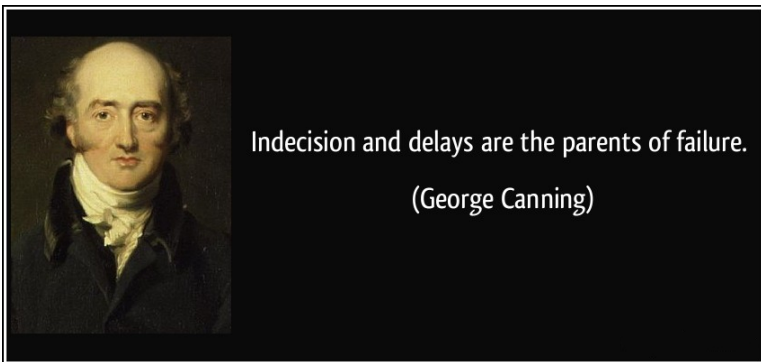
---

<sup>\*</sup> William Shakespeare, Twelfth Night/What You Will, Act 2, Scene 3.

## A Pearl of Wisdom



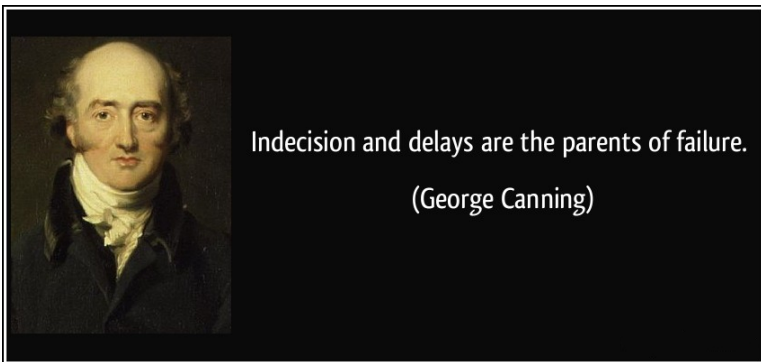
©izQuotes



©izQuotes

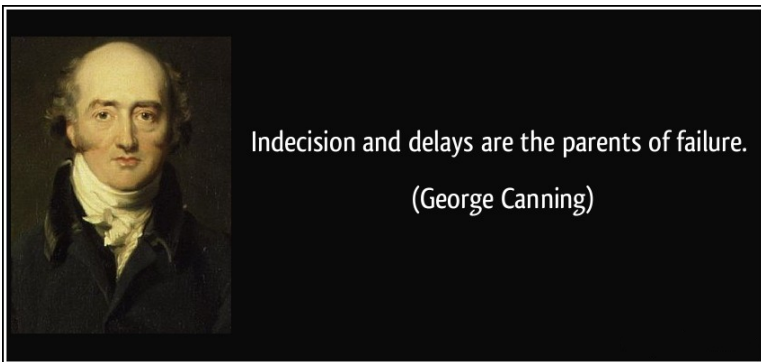
- Only relevant to ordinary people's life?

## A Pearl of Wisdom



©izQuotes

- Only relevant to ordinary people's life?
- Or to scientists, in particular **comp. sci.** and **control folks**, too?



©izQuotes

- Only relevant to ordinary people's life?
- Or to scientists, in particular **comp. sci.** and **control folks**, too?

Remember that Canning briefly **controlled** Great Britain!

# Staying Safe

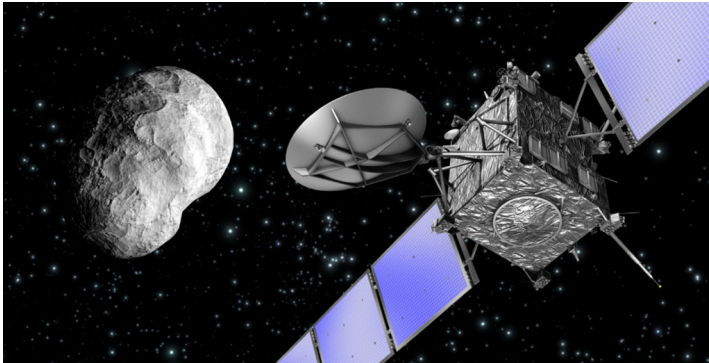
When Observation & Actuation Suffer from Serious Delays



©ESA

# Staying Safe

When Observation & Actuation Suffer from Serious Delays



©ESA

- You could move slowly. (Well, can you?)

# Staying Safe

When Observation & Actuation Suffer from Serious Delays



©ESA

- You could move slowly. (Well, can you?)
- You could trust autonomy.



# Staying Safe

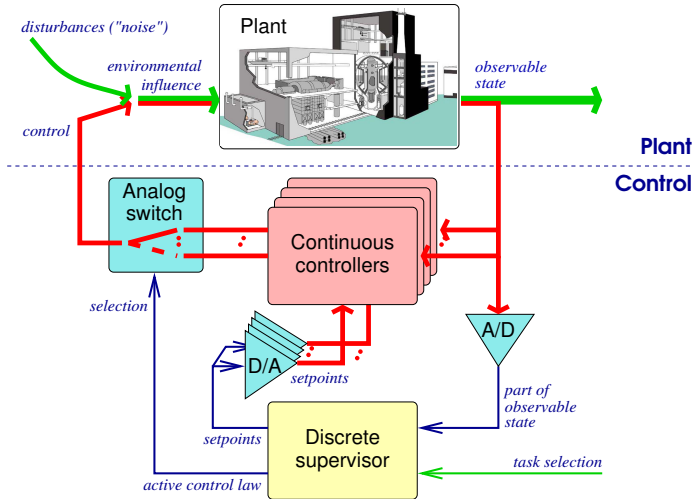
When Observation & Actuation Suffer from Serious Delays



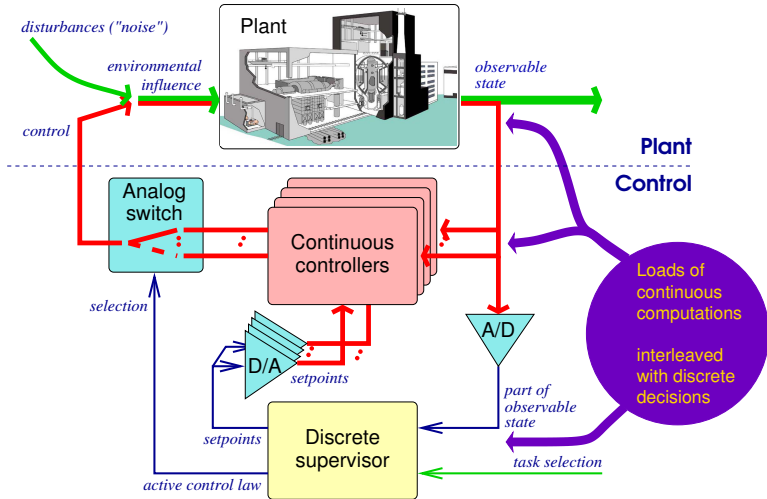
©ESA

- You could move slowly. (Well, can you?)
- You could trust autonomy.
- Or you have to **anticipate** and **issue actions early**.

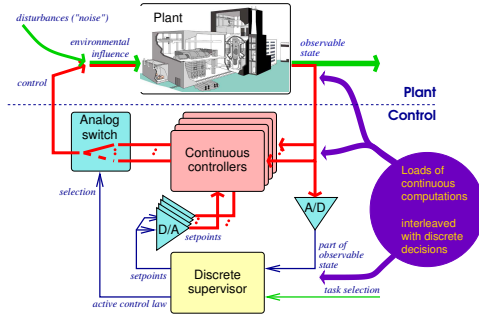
# Interaction btw. a Controller and Its Environment



# Interaction btw. a Controller and Its Environment



# Interaction btw. a Controller and Its Environment

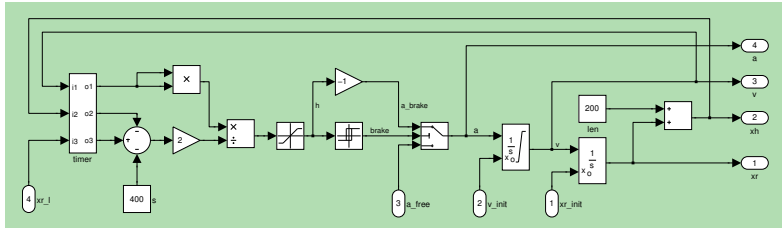


**Crucial question :** How do the controller and the plant interact?

**Traditional answer :** Coupling assumed to be (or at least modeled as) **delay-free** :

- **mode dynamics** is covered by the **conjunction of individual ODEs**;
- **switching btw. modes** is an **immediate reaction to environmental conditions**.

# Instantaneous Coupling

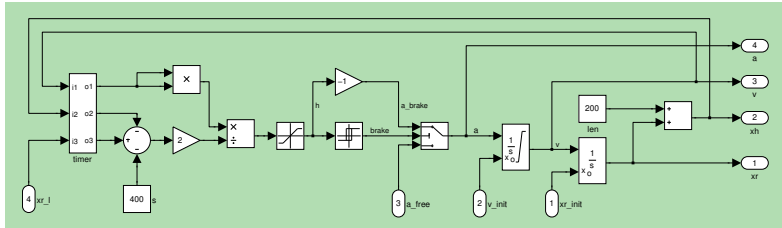


©ETCS-3

Following the tradition, the above (rather typical) Simulink model assumes

- **delay-free coupling** between all components;
- **instantaneous feed-through** within all functional blocks.

# Instantaneous Coupling



©ETCS-3

Following the tradition, the above (rather typical) Simulink model assumes

- delay-free coupling between all components;
- instantaneous feed-through within all functional blocks.

Central questions :

- 1 Is this **realistic**?
- 2 If not, does it have **observable effects on control performance**?
- 3 May those effects be **detrimental or even harmful**?

## Q1 : Is Instantaneous Coupling Realistic?



## Q1 : Is Instantaneous Coupling Realistic?

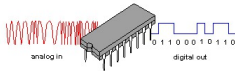


**We are no better :**

As soon as computer scientists enter the scene, serious delays are ahead ...



# Q1 : Is Instantaneous Coupling Realistic?



Digital control needs **A/D and D/A conversion**, which induces **latency in signal forwarding**.



Digital **signal processing**, especially in complex sensors like CV, needs **processing time**, adding signal delays.

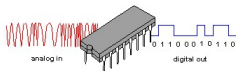


**Networked control** introduces **communication latency** into the feedback control loop.



Harvesting, fusing, and forwarding data through **sensor networks** **enlarge the communication latency** by orders of magnitude.

# Q1 : Is Instantaneous Coupling Realistic? – No.



Digital control needs **A/D and D/A conversion**, which induces **latency in signal forwarding**.



Digital signal processing is especially in complex sensors, adding signal delays.



communication latency



Harvesting, fusing, and forwarding data through **sensor networks** **enlarge the communication latency** by orders of magnitude.

## Q2 : Do Delays Have Observable Effects?

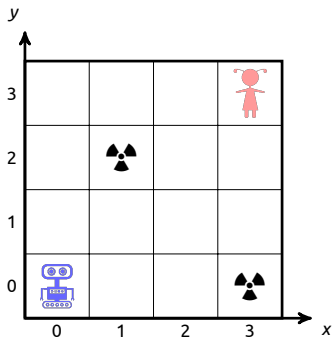


Figure – A robot escape game in a  $4 \times 4$  room.

## Q2 : Do Delays Have Observable Effects?

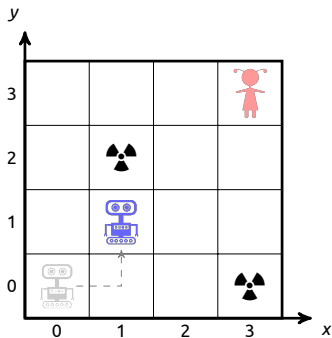


Figure – A robot escape game in a  $4 \times 4$  room.

## Q2 : Do Delays Have Observable Effects?

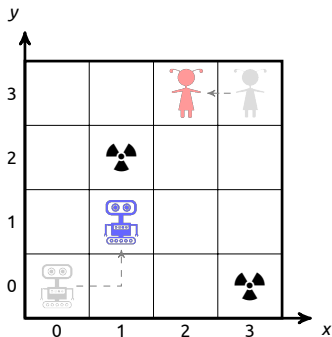
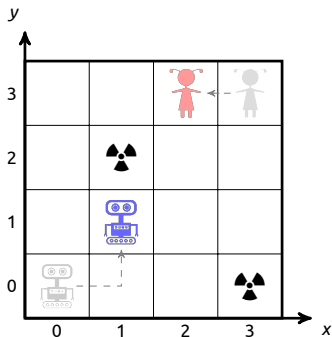


Figure – A robot escape game in a  $4 \times 4$  room.

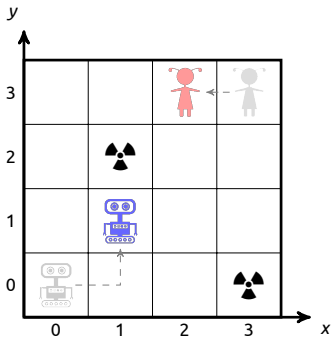
## Q2 : Do Delays Have Observable Effects?



No delay :

Figure – A robot escape game in a  $4 \times 4$  room.

## Q2 : Do Delays Have Observable Effects?



No delay :

Robot always wins by circling around the obstacle ☢ at (1,2).

Figure – A robot escape game in a  $4 \times 4$  room.

## Q2 : Do Delays Have Observable Effects?

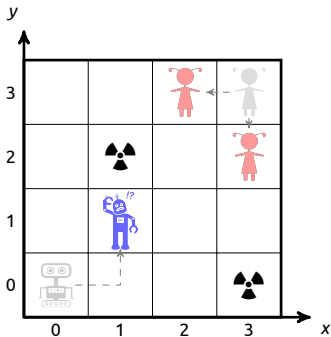


Figure – A robot escape game in a  $4 \times 4$  room.

No delay :

Robot always wins by circling around the obstacle ☢ at (1,2).

1 step delay :



## Q2 : Do Delays Have Observable Effects?

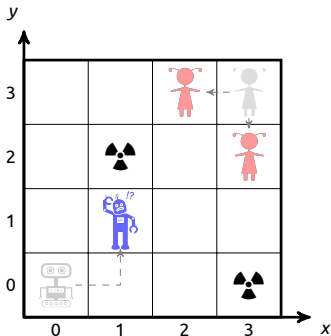


Figure – A robot escape game in a  $4 \times 4$  room.

No delay :

Robot always wins by circling around the obstacle ♻ at (1,2).

1 step delay :

Robot wins by 1-step pre-decision.

## Q2 : Do Delays Have Observable Effects?

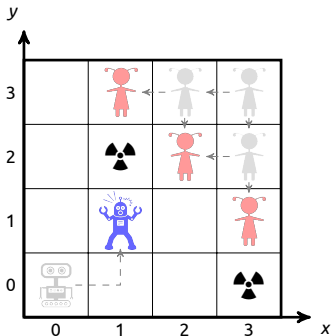



Figure – A robot escape game in a  $4 \times 4$  room.

No delay :

Robot always wins by circling around the obstacle  at (1,2).

1 step delay :

Robot wins by 1-step pre-decision.

2 steps delay :

## Q2 : Do Delays Have Observable Effects?

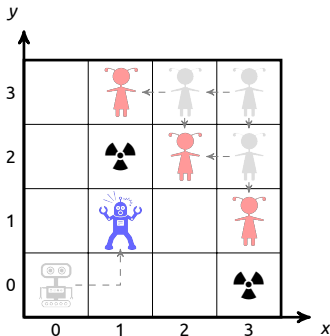


Figure – A robot escape game in a  $4 \times 4$  room.

### No delay :

Robot always wins by circling around the obstacle ☢ at (1,2).

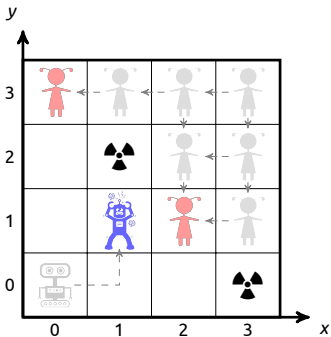
### 1 step delay :

Robot wins by 1-step pre-decision.

### 2 steps delay :

Robot still wins, yet **extra memory** is needed.

## Q2 : Do Delays Have Observable Effects?



No delay :

Robot always wins by circling around the obstacle 🚧 at (1,2).

1 step delay :

Robot wins by 1-step pre-decision.

2 steps delay :

Robot still wins, yet **extra memory** is needed.

3 steps delay :

**Figure** – A robot escape game in a  $4 \times 4$  room.

## Q2 : Do Delays Have Observable Effects?

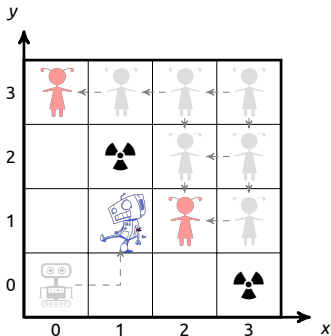


Figure – A robot escape game in a  $4 \times 4$  room.

### No delay :

Robot always wins by circling around the obstacle ☢ at (1,2).

### 1 step delay :

Robot wins by 1-step pre-decision.

### 2 steps delay :

Robot still wins, yet **extra memory** is needed.

### 3 steps delay :

Robot is unwinnable (**uncontrollable**) anymore.

## Q2 : Do Delays Have Observable Effects? – Yes, they have.

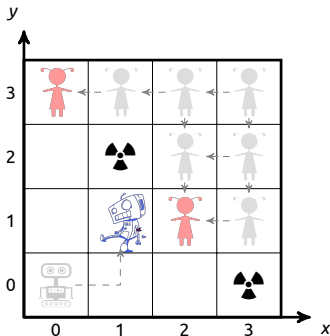


Figure – A robot escape game in a  $4 \times 4$  room.

### No delay :

Robot always wins by circling around the obstacle ☢ at (1,2).

### 1 step delay :

Robot wins by 1-step pre-decision.

### 2 steps delay :

Robot still wins, yet **extra memory** is needed.

### 3 steps delay :

Robot is unwinnable (**uncontrollable**) anymore.

### Q3 : May the Effects be Harmful? – Yes, delays may well annihilate the control performance.

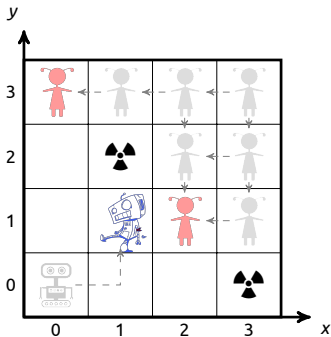


Figure – A robot escape game in a  $4 \times 4$  room.

#### No delay :

Robot always wins by circling around the obstacle  $\blacktriangle$  at (1,2).

#### 1 step delay :

Robot wins by 1-step pre-decision.

#### 2 steps delay :

Robot still wins, yet **extra memory** is needed.

#### 3 steps delay :

Robot is unwinnable (**uncontrollable**) anymore.

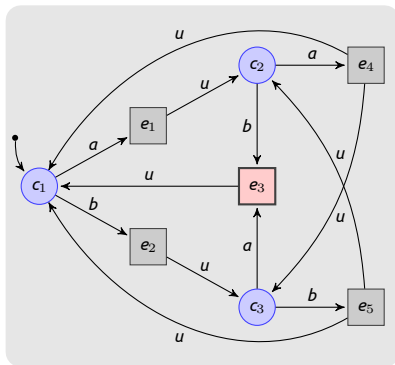
# Outline

- 1 Safety Games under Delayed Interaction
- 2 Synthesizing Delay-Resilient Control
- 3 Extended Delay Patterns



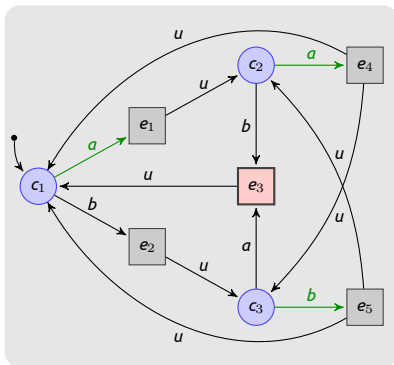


# A Trivial Safety Game



Goal: Avoid e<sub>3</sub> by appropriate actions of player c.

# A Trivial Safety Game



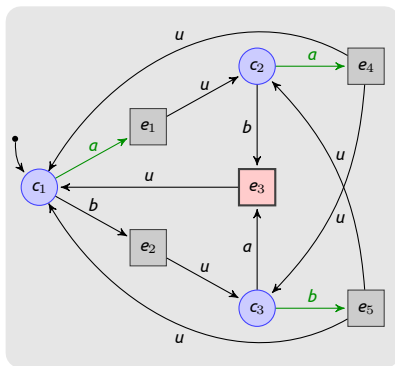
**Goal :** Avoid  $e_3$  by appropriate actions of player  $c$ .

**Strategy :** May always play  $a$  except in  $c_3$  :

$$c_1, c_2 \mapsto a$$

$$c_3 \mapsto b$$

# A Trivial Safety Game



**Goal :** Avoid  $e_3$  by appropriate actions of player  $c$ .

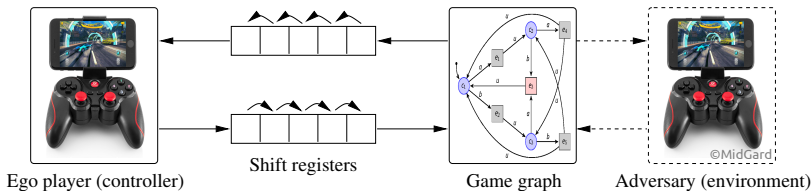
**Strategy :** May always play  $a$  except in  $c_3$  :

$$c_1, c_2 \mapsto a$$

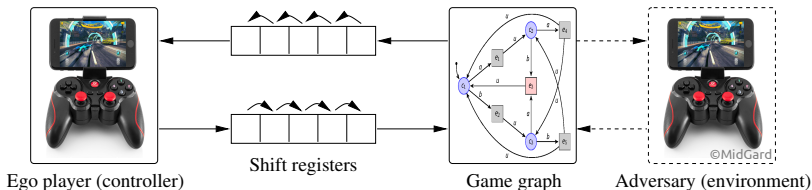
$$c_3 \mapsto b$$

**Properties :** Determinacy and memoryless.

## Playing Safety Games under Discrete Delay

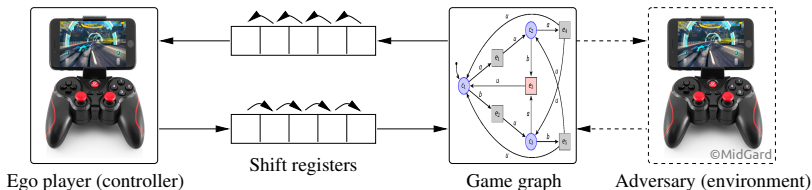


# Playing Safety Games under Discrete Delay



**Observation :** It doesn't make an observable difference for the joint dynamics whether delay occurs in *perception*, *actuation*, or *both*.

# Playing Safety Games under Discrete Delay

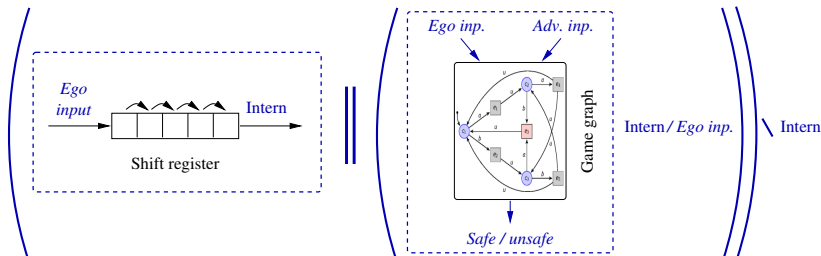


**Observation :** It doesn't make an observable difference for the joint dynamics whether delay occurs in *perception*, *actuation*, or *both*.

**Consequence :** An obvious *reduction* to a safety game of *perfect information*.

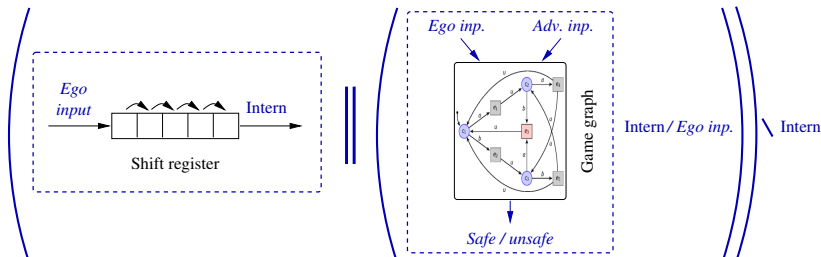
# Reduction to Delay-Free Games

from Ego-Player Perspective



# Reduction to Delay-Free Games

from Ego-Player Perspective



- 😊 Safety games w. delay **can be solved algorithmically**.
- 😞 Game graph incurs **blow-up by factor  $|\text{Alphabet}(\text{ego})|^{\text{delay}}$** .



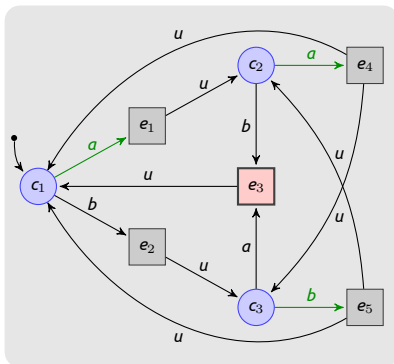
# The Simple Safety Game

... but with Delay

No delay :

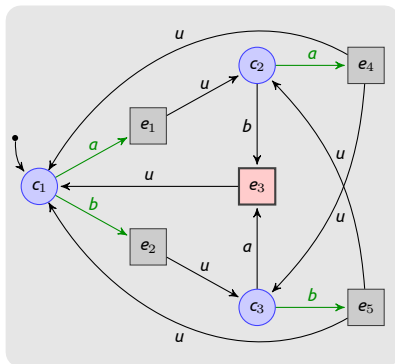
$c_1, c_2 \mapsto a$

$c_3 \mapsto b$



# The Simple Safety Game

... but with Delay



No delay :

$c_1, c_2 \mapsto a$

$c_3 \mapsto b$

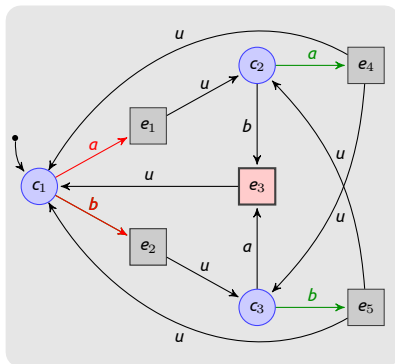
1 step delay :

$e_1, e_5 \mapsto a$

$e_2, e_4 \mapsto b$

# The Simple Safety Game

... but with Delay



No delay :

$$\begin{aligned} c_1, c_2 &\mapsto a \\ c_3 &\mapsto b \end{aligned}$$

1 step delay :

$$\begin{aligned} e_1, e_5 &\mapsto a \\ e_2, e_4 &\mapsto b \end{aligned}$$

2 steps delay :

$$c_1 \mapsto \begin{cases} a & \text{if 2 steps back} \\ & \text{an } a \text{ was issued,} \\ b & \text{if 2 steps back} \\ & \text{a } b \text{ was issued.} \end{cases}$$

$$\begin{aligned} c_2 &\mapsto b \\ c_3 &\mapsto a \end{aligned}$$

Need memory!

# Incremental Synthesis in a Nutshell

- Observation :** A winning strategy for delay  $k' > k$  can always be utilized for a safe win under delay  $k$ .
- Consequence :** A position is winning for delay  $k$  is a necessary condition for it being winning under delay  $k' > k$ .

⇒ M. Chen, M. Fränzle, Y. Li, P. N. Mosaad, N. Zhan : *What's to come is still unsure : Synthesizing controllers resilient to delayed interaction*. ATVA '18. [Distinguished Paper Award].

# Incremental Synthesis in a Nutshell

**Observation :** A winning strategy for delay  $k' > k$  can always be utilized for a safe win under delay  $k$ .

**Consequence :** A position is winning for delay  $k$  is a necessary condition for it being winning under delay  $k' > k$ .

**Idea :** Incrementally filter out loss states & incrementally synthesize winning strategy for the remaining :

- 1 synthesize winning strategy for the *delay-free* counterpart;
- 2 for each winning state, *lift strategy from delay  $k$  to  $k + 1$* ;
- 3 *remove states* where this does not succeed;
- 4 repeat from 2 until either delay-resilience suffices (*winning*) or initial state turns lossy (*losing*).

⇒ M. Chen, M. Fränzle, Y. Li, P. N. Mosaad, N. Zhan : *What's to come is still unsure : Synthesizing controllers resilient to delayed interaction*. ATVA '18. [Distinguished Paper Award].

# Incremental Synthesis in a Nutshell

**Observation :** A winning strategy for delay  $k' > k$  can always be utilized for a safe win under delay  $k$ .

**Consequence :** A position is winning for delay  $k$  is a necessary condition for it being winning under delay  $k' > k$ .

**Idea :** Incrementally filter out loss states & incrementally synthesize winning strategy for the remaining :

- 1 synthesize winning strategy for the *delay-free* counterpart;
- 2 for each winning state, *lift strategy from delay  $k$  to  $k + 1$* ;
- 3 *remove states* where this does not succeed;
- 4 repeat from 2 until either delay-resilience suffices (*winning*) or initial state turns lossy (*losing*).

⇒ M. Chen, M. Fränzle, Y. Li, P. N. Mosaad, N. Zhan : *What's to come is still unsure : Synthesizing controllers resilient to delayed interaction*. ATVA '18. [Distinguished Paper Award].

# Incremental Synthesis in a Nutshell

**Observation :** A winning strategy for delay  $k' > k$  can always be utilized for a safe win under delay  $k$ .

**Consequence :** A position is winning for delay  $k$  is a necessary condition for it being winning under delay  $k' > k$ .

**Idea :** Incrementally filter out loss states & incrementally synthesize winning strategy for the remaining :

- 1 synthesize winning strategy for the *delay-free* counterpart;
- 2 for each winning state, *lift strategy from delay  $k$  to  $k + 1$* ;
- 3 *remove states* where this does not succeed;
- 4 repeat from 2 until either delay-resilience suffices (*winning*) or initial state turns lossy (*losing*).

⇒ M. Chen, M. Fränzle, Y. Li, P. N. Mosaad, N. Zhan : *What's to come is still unsure : Synthesizing controllers resilient to delayed interaction*. ATVA '18. [Distinguished Paper Award].

# Incremental Synthesis in a Nutshell

**Observation :** A winning strategy for delay  $k' > k$  can always be utilized for a safe win under delay  $k$ .

**Consequence :** A position is winning for delay  $k$  is a necessary condition for it being winning under delay  $k' > k$ .

**Idea :** Incrementally filter out loss states & incrementally synthesize winning strategy for the remaining :

- 1 synthesize winning strategy for the *delay-free* counterpart;
- 2 for each winning state, *lift strategy from delay  $k$  to  $k + 1$* ;
- 3 *remove states* where this does not succeed;
- 4 repeat from 2 until either delay-resilience suffices (*winning*) or initial state turns lossy (*losing*).

⇒ M. Chen, M. Fränzle, Y. Li, P. N. Mosaad, N. Zhan : *What's to come is still unsure : Synthesizing controllers resilient to delayed interaction*. ATVA '18. [Distinguished Paper Award].



# Incremental Synthesis in a Nutshell

**Observation :** A winning strategy for delay  $k' > k$  can always be utilized for a safe win under delay  $k$ .

**Consequence :** A position is winning for delay  $k$  is a necessary condition for it being winning under delay  $k' > k$ .

**Idea :** Incrementally filter out loss states & incrementally synthesize winning strategy for the remaining :

- 1 synthesize winning strategy for the *delay-free* counterpart;
- 2 for each winning state, *lift strategy from delay  $k$  to  $k + 1$* ;
- 3 *remove states* where this does not succeed;
- 4 repeat from 2 until either delay-resilience suffices (*winning*) or initial state turns lossy (*losing*).

⇒ M. Chen, M. Fränzle, Y. Li, P. N. Mosaad, N. Zhan : *What's to come is still unsure : Synthesizing controllers resilient to delayed interaction*. ATVA '18. [Distinguished Paper Award].

# Incremental Synthesis of Delay-Tolerant Strategies

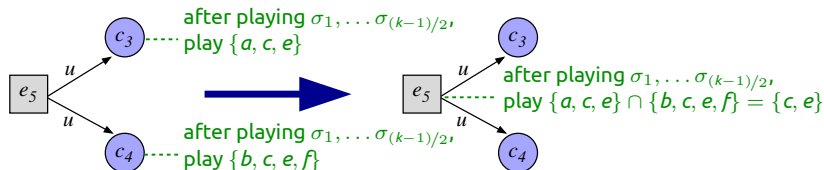
- 1 Generate a *maximally permissive* strategy for delay  $k = 0$ .

# Incremental Synthesis of Delay-Tolerant Strategies

1 Generate a *maximally permissive* strategy for delay  $k = 0$ .

2 Advance to delay  $k + 1$  :

If  $k$  odd : For each (ego-)winning adversarial state define strategy as



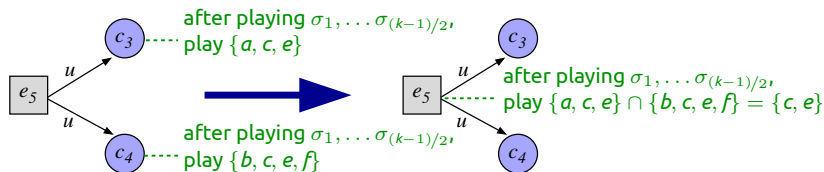
... and eliminate any dead ends by bwd. traversal.

# Incremental Synthesis of Delay-Tolerant Strategies

1 Generate a *maximally permissive* strategy for delay  $k = 0$ .

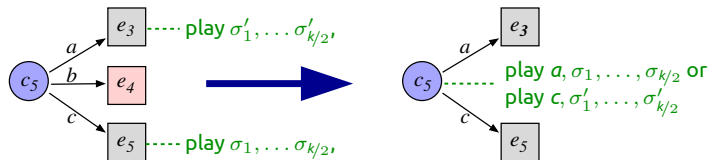
2 Advance to delay  $k + 1$  :

If  $k$  odd : For each (ego-)winning adversarial state define strategy as



... and eliminate any dead ends by bwd. traversal.

If  $k$  even : For each winning ego state define strategy as

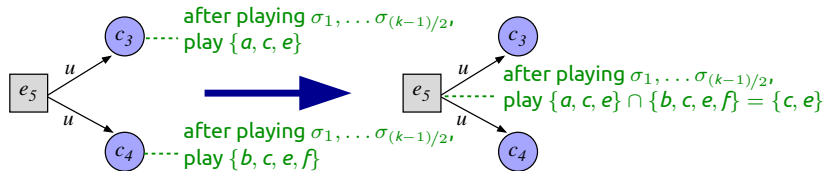


# Incremental Synthesis of Delay-Tolerant Strategies

1 Generate a *maximally permissive* strategy for delay  $k = 0$ .

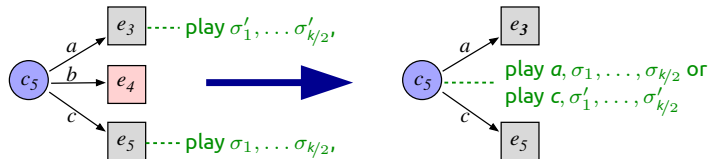
2 Advance to delay  $k + 1$  :

If  $k$  odd : For each (ego-)winning adversarial state define strategy as



... and eliminate any dead ends by bwd. traversal.

If  $k$  even : For each winning ego state define strategy as



3 Repeat from 2 until either delay-resilience suffices or initial state turns lossy.

# Incremental- vs. Reduction-Based Synthesis

| Benchmark |      |       |    | Reduction + Explicit-State Synthesis |              |              |              |              |              | Incremental Explicit-State Synthesis |              |              |              |               |              |       |
|-----------|------|-------|----|--------------------------------------|--------------|--------------|--------------|--------------|--------------|--------------------------------------|--------------|--------------|--------------|---------------|--------------|-------|
| name      | S    | →     | U  | $\delta_{\max}$                      | $\delta = 0$ | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta_{\max}$                      | $\delta = 0$ | $\delta = 1$ | $\delta = 2$ | $\delta = 3$  | $\delta = 4$ | %     |
| Exmp.trv1 | 14   | 20    | 4  | $\geq 22$                            | 0.00         | 0.00         | 0.01         | 0.02         | 0.02         | $\geq 30$                            | 0.00         | 0.00         | <b>0.00</b>  | <b>0.01</b>   | <b>0.01</b>  | –     |
| Exmp.trv2 | 14   | 22    | 4  | $= 2$                                | 0.00         | 0.01         | 0.01         | 0.02         | –            | $= 2$                                | 0.00         | <b>0.00</b>  | <b>0.00</b>  | <b>0.01</b>   | –            | 81.97 |
| Escp.4×4  | 224  | 738   | 16 | $= 2$                                | 0.08         | 11.66        | 11.73        | 1059.23      | –            | $= 2$                                | 0.08         | <b>0.13</b>  | <b>0.22</b>  | <b>0.25</b>   | –            | 99.02 |
| Escp.4×5  | 360  | 1326  | 20 | $= 2$                                | 0.18         | 34.09        | 33.80        | 3084.58      | –            | $= 2$                                | 0.18         | <b>0.27</b>  | <b>0.46</b>  | <b>0.63</b>   | –            | 99.02 |
| Escp.5×5  | 598  | 2301  | 26 | $\geq 2$                             | 0.46         | 96.24        | 97.10        | ?            | ?            | $= 2$                                | 0.46         | <b>0.68</b>  | <b>1.16</b>  | <b>1.71</b>   | –            | 98.98 |
| Escp.5×6  | 840  | 3516  | 30 | $\geq 2$                             | 1.01         | 217.63       | 216.83       | ?            | ?            | $= 2$                                | <b>1.00</b>  | <b>1.42</b>  | <b>2.40</b>  | <b>4.30</b>   | –            | 99.00 |
| Escp.6×6  | 1224 | 5424  | 36 | $\geq 2$                             | 2.13         | 516.92       | 511.41       | ?            | ?            | $= 2$                                | <b>2.06</b>  | <b>2.90</b>  | <b>5.12</b>  | <b>10.30</b>  | –            | 98.97 |
| Escp.7×7  | 2350 | 11097 | 50 | $\geq 2$                             | 7.81         | 2167.86      | 2183.01      | ?            | ?            | $= 2$                                | <b>7.71</b>  | <b>10.67</b> | <b>19.04</b> | <b>52.47</b>  | –            | 98.99 |
| Escp.7×8  | 3024 | 14820 | 56 | $\geq 0$                             | <b>13.07</b> | ?            | ?            | ?            | ?            | $= 2$                                | 13.44        | <b>18.25</b> | <b>32.69</b> | <b>108.60</b> | –            | 99.01 |

| Benchmark    |                 | Reduction + Yosys + SafetySynth (symbolic) |              |              |              |              |              |              | Incremental Synthesis (explicit-state implementation) |              |              |              |              |              |              |       |
|--------------|-----------------|--|--------------|--------------|--------------|--------------|--------------|--------------|---|--------------|--------------|--------------|--------------|--------------|--------------|-------|
| name         | $\delta_{\max}$ | $\delta = 0$                               | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta = 5$ | $\delta = 6$ | $\delta = 0$  | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta = 5$ | $\delta = 6$ | %     |
| Stub.4x4 = 2 |                 | 1.07                                       | 1.24         | 1.24         | 1.80         | –            | –            | –            | <b>0.04</b>   | <b>0.07</b>  | <b>0.12</b>  | <b>0.18</b>  | –            | –            | –            | 98.98 |
| Stub.4x5 = 2 |                 | 1.16                                       | 1.49         | 1.49         | 2.83         | –            | –            | –            | <b>0.08</b>   | <b>0.14</b>  | <b>0.25</b>  | <b>0.44</b>  | –            | –            | –            | 98.97 |
| Stub.5x5 = 2 |                 | 1.19                                       | 2.61         | 2.50         | 13.67        | –            | –            | –            | <b>0.21</b>   | <b>0.37</b>  | <b>0.63</b>  | <b>1.17</b>  | –            | –            | –            | 98.97 |
| Stub.5x6 = 2 |                 | 1.18                                       | 2.60         | 2.59         | 23.30        | –            | –            | –            | <b>0.42</b>   | <b>0.69</b>  | <b>1.20</b>  | <b>2.49</b>  | –            | –            | –            | 98.96 |
| Stub.6x6 = 4 |                 | 1.17                                       | 2.76         | 2.74         | 19.96        | 19.69        | 655.24       | –            | <b>0.93</b>   | <b>1.47</b>  | <b>2.60</b>  | <b>5.79</b>  | <b>7.54</b>  | <b>7.60</b>  | –            | 99.89 |
| Stub.7x7 = 4 |                 | <b>1.23</b>                                | <b>2.50</b>  | <b>2.48</b>  | 24.57        | <b>23.01</b> | 2224.62      | –            | 3.60  | 5.52         | 10.08        | <b>22.75</b> | 31.18        | <b>32.98</b> | –            | 99.88 |

# Incremental- vs. Reduction-Based Synthesis

| Benchmark |      |       |    | Reduction + Explicit-State Synthesis |              |              |              |              |              | Incremental Explicit-State Synthesis |              |              |              |               |              |       |  |
|-----------|------|-------|----|--------------------------------------|--------------|--------------|--------------|--------------|--------------|--------------------------------------|--------------|--------------|--------------|---------------|--------------|-------|--|
| name      | S    | →     | U  | $\delta_{\max}$                      | $\delta = 0$ | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta_{\max}$                      | $\delta = 0$ | $\delta = 1$ | $\delta = 2$ | $\delta = 3$  | $\delta = 4$ | %     |  |
| Exmp.trv1 | 14   | 20    | 4  | $\geq 22$                            | 0.00         | 0.00         | 0.01         | 0.02         | 0.02         | $\geq 30$                            | 0.00         | 0.00         | <b>0.00</b>  | <b>0.01</b>   | <b>0.01</b>  | –     |  |
| Exmp.trv2 | 14   | 22    | 4  | $= 2$                                | 0.00         | 0.01         | 0.01         | 0.02         | –            | $= 2$                                | 0.00         | <b>0.00</b>  | <b>0.00</b>  | <b>0.01</b>   | –            | 81.97 |  |
| Escp.4x4  | 224  | 738   | 16 | $= 2$                                | 0.08         | 11.66        | 11.73        | 1059.23      | –            | $= 2$                                | 0.08         | <b>0.13</b>  | <b>0.22</b>  | <b>0.25</b>   | –            | 99.02 |  |
| Escp.4x5  | 360  | 1326  | 20 | $= 2$                                | 0.18         | 34.09        | 33.80        | 3084.58      | –            | $= 2$                                | 0.18         | <b>0.27</b>  | <b>0.46</b>  | <b>0.63</b>   | –            | 99.02 |  |
| Escp.5x5  | 598  | 2301  | 26 | $\geq 2$                             | 0.46         | 96.24        | 97.10        | ?            | ?            | $= 2$                                | 0.46         | <b>0.68</b>  | <b>1.16</b>  | <b>1.71</b>   | –            | 98.98 |  |
| Escp.5x6  | 840  | 3516  | 30 | $\geq 2$                             | 1.01         | 217.63       | 216.83       | ?            | ?            | $= 2$                                | <b>1.00</b>  | <b>1.42</b>  | <b>2.40</b>  | <b>4.30</b>   | –            | 99.00 |  |
| Escp.6x6  | 1224 | 5424  | 36 | $\geq 2$                             | 2.13         | 516.92       | 511.41       | ?            | ?            | $= 2$                                | <b>2.06</b>  | <b>2.90</b>  | <b>5.12</b>  | <b>10.30</b>  | –            | 98.97 |  |
| Escp.7x7  | 2350 | 11097 | 50 | $\geq 2$                             | 7.81         | 2167.86      | 2183.01      | ?            | ?            | $= 2$                                | <b>7.71</b>  | <b>10.67</b> | <b>19.04</b> | <b>52.47</b>  | –            | 98.99 |  |
| Escp.7x8  | 3024 | 14820 | 56 | $\geq 0$                             | <b>13.07</b> | ?            | ?            | ?            | ?            | $= 2$                                | 13.44        | <b>18.25</b> | <b>32.69</b> | <b>108.60</b> | –            | 99.01 |  |

| Benchmark    |                 | Reduction + Yosys + SafetySynth (symbolic) |              |              |              |              |              |              | Incremental Synthesis (explicit-state implementation) |              |              |              |              |              |              |       |
|--------------|-----------------|--|--------------|--------------|--------------|--------------|--------------|--------------|---|--------------|--------------|--------------|--------------|--------------|--------------|-------|
| name         | $\delta_{\max}$ | $\delta = 0$                               | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta = 5$ | $\delta = 6$ | $\delta = 0$  | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta = 5$ | $\delta = 6$ | %     |
| Stub.4x4 = 2 |                 | 1.07                                       | 1.24         | 1.24         | 1.80         | –            | –            | –            | <b>0.04</b>   | <b>0.07</b>  | <b>0.12</b>  | <b>0.18</b>  | –            | –            | –            | 98.98 |
| Stub.4x5 = 2 |                 | 1.16                                       | 1.49         | 1.49         | 2.83         | –            | –            | –            | <b>0.08</b>   | <b>0.14</b>  | <b>0.25</b>  | <b>0.44</b>  | –            | –            | –            | 98.97 |
| Stub.5x5 = 2 |                 | 1.19                                       | 2.61         | 2.50         | 13.67        | –            | –            | –            | <b>0.21</b>   | <b>0.37</b>  | <b>0.63</b>  | <b>1.17</b>  | –            | –            | –            | 98.97 |
| Stub.5x6 = 2 |                 | 1.18                                       | 2.60         | 2.59         | 23.30        | –            | –            | –            | <b>0.42</b>   | <b>0.69</b>  | <b>1.20</b>  | <b>2.49</b>  | –            | –            | –            | 98.96 |
| Stub.6x6 = 4 |                 | 1.17                                       | 2.76         | 2.74         | 19.96        | 19.69        | 655.24       | –            | <b>0.93</b>   | <b>1.47</b>  | <b>2.60</b>  | <b>5.79</b>  | <b>7.54</b>  | <b>7.60</b>  | –            | 99.89 |
| Stub.7x7 = 4 |                 | <b>1.23</b>                                | <b>2.50</b>  | <b>2.48</b>  | 24.57        | <b>23.01</b> | 2224.62      | –            | 3.60  | 5.52         | 10.08        | <b>22.75</b> | 31.18        | <b>32.98</b> | –            | 99.88 |

# Incremental- vs. Reduction-Based Synthesis

| Benchmark |      |       |    | Reduction + Explicit-State Synthesis |              |              |              |              |              | Incremental Explicit-State Synthesis |              |              |              |               |              |       |  |
|-----------|------|-------|----|--------------------------------------|--------------|--------------|--------------|--------------|--------------|--------------------------------------|--------------|--------------|--------------|---------------|--------------|-------|--|
| name      | S    | →     | U  | $\delta_{\max}$                      | $\delta = 0$ | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta_{\max}$                      | $\delta = 0$ | $\delta = 1$ | $\delta = 2$ | $\delta = 3$  | $\delta = 4$ | %     |  |
| Exmp.trv1 | 14   | 20    | 4  | $\geq 22$                            | 0.00         | 0.00         | 0.01         | 0.02         | 0.02         | $\geq 30$                            | 0.00         | 0.00         | <b>0.00</b>  | <b>0.01</b>   | <b>0.01</b>  | –     |  |
| Exmp.trv2 | 14   | 22    | 4  | $= 2$                                | 0.00         | 0.01         | 0.01         | 0.02         | –            | $= 2$                                | 0.00         | <b>0.00</b>  | <b>0.00</b>  | <b>0.01</b>   | –            | 81.97 |  |
| Escp.4x4  | 224  | 738   | 16 | $= 2$                                | 0.08         | 11.66        | 11.73        | 1059.23      | –            | $= 2$                                | 0.08         | <b>0.13</b>  | <b>0.22</b>  | <b>0.25</b>   | –            | 99.02 |  |
| Escp.4x5  | 360  | 1326  | 20 | $= 2$                                | 0.18         | 34.09        | 33.80        | 3084.58      | –            | $= 2$                                | 0.18         | <b>0.27</b>  | <b>0.46</b>  | <b>0.63</b>   | –            | 99.02 |  |
| Escp.5x5  | 598  | 2301  | 26 | $\geq 2$                             | 0.46         | 96.24        | 97.10        | ?            | ?            | $= 2$                                | 0.46         | <b>0.68</b>  | <b>1.16</b>  | <b>1.71</b>   | –            | 98.98 |  |
| Escp.5x6  | 840  | 3516  | 30 | $\geq 2$                             | 1.01         | 217.63       | 216.83       | ?            | ?            | $= 2$                                | <b>1.00</b>  | <b>1.42</b>  | <b>2.40</b>  | <b>4.30</b>   | –            | 99.00 |  |
| Escp.6x6  | 1224 | 5424  | 36 | $\geq 2$                             | 2.13         | 516.92       | 511.41       | ?            | ?            | $= 2$                                | <b>2.06</b>  | <b>2.90</b>  | <b>5.12</b>  | <b>10.30</b>  | –            | 98.97 |  |
| Escp.7x7  | 2350 | 11097 | 50 | $\geq 2$                             | 7.81         | 2167.86      | 2183.01      | ?            | ?            | $= 2$                                | <b>7.71</b>  | <b>10.67</b> | <b>19.04</b> | <b>52.47</b>  | –            | 98.99 |  |
| Escp.7x8  | 3024 | 14820 | 56 | $\geq 0$                             | <b>13.07</b> | ?            | ?            | ?            | ?            | $= 2$                                | 13.44        | <b>18.25</b> | <b>32.69</b> | <b>108.60</b> | –            | 99.01 |  |

| Benchmark    |                 | Reduction + Yosys + SafetySynth (symbolic) |              |              |              |              |              |              | Incremental Synthesis (explicit-state implementation) |              |              |              |              |              |              |       |
|--------------|-----------------|--|--------------|--------------|--------------|--------------|--------------|--------------|---|--------------|--------------|--------------|--------------|--------------|--------------|-------|
| name         | $\delta_{\max}$ | $\delta = 0$                               | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta = 5$ | $\delta = 6$ | $\delta = 0$  | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta = 5$ | $\delta = 6$ | %     |
| Stub.4x4 = 2 |                 | 1.07                                       | 1.24         | 1.24         | 1.80         | –            | –            | –            | <b>0.04</b>   | <b>0.07</b>  | <b>0.12</b>  | <b>0.18</b>  | –            | –            | –            | 98.98 |
| Stub.4x5 = 2 |                 | 1.16                                       | 1.49         | 1.49         | 2.83         | –            | –            | –            | <b>0.08</b>   | <b>0.14</b>  | <b>0.25</b>  | <b>0.44</b>  | –            | –            | –            | 98.97 |
| Stub.5x5 = 2 |                 | 1.19                                       | 2.61         | 2.50         | 13.67        | –            | –            | –            | <b>0.21</b>   | <b>0.37</b>  | <b>0.63</b>  | <b>1.17</b>  | –            | –            | –            | 98.97 |
| Stub.5x6 = 2 |                 | 1.18                                       | 2.60         | 2.59         | 23.30        | –            | –            | –            | <b>0.42</b>   | <b>0.69</b>  | <b>1.20</b>  | <b>2.49</b>  | –            | –            | –            | 98.96 |
| Stub.6x6 = 4 |                 | 1.17                                       | 2.76         | 2.74         | 19.96        | 19.69        | 655.24       | –            | <b>0.93</b>   | <b>1.47</b>  | <b>2.60</b>  | <b>5.79</b>  | <b>7.54</b>  | <b>7.60</b>  | –            | 99.89 |
| Stub.7x7 = 4 |                 | <b>1.23</b>                                | <b>2.50</b>  | <b>2.48</b>  | 24.57        | <b>23.01</b> | 2224.62      | –            | 3.60  | 5.52         | 10.08        | <b>22.75</b> | 31.18        | <b>32.98</b> | –            | 99.88 |



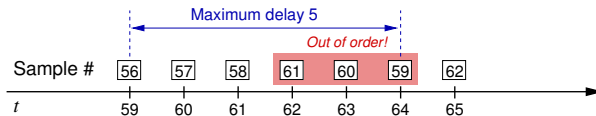
# Incremental- vs. Reduction-Based Synthesis

| Benchmark |      |       |    | Reduction + Explicit-State Synthesis |              |              |              |              |              | Incremental Explicit-State Synthesis |              |              |              |               |              |       |  |
|-----------|------|-------|----|--------------------------------------|--------------|--------------|--------------|--------------|--------------|--------------------------------------|--------------|--------------|--------------|---------------|--------------|-------|--|
| name      | S    | →     | U  | $\delta_{\max}$                      | $\delta = 0$ | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta_{\max}$                      | $\delta = 0$ | $\delta = 1$ | $\delta = 2$ | $\delta = 3$  | $\delta = 4$ | %     |  |
| Exmp.trv1 | 14   | 20    | 4  | $\geq 22$                            | 0.00         | 0.00         | 0.01         | 0.02         | 0.02         | $\geq 30$                            | 0.00         | 0.00         | <b>0.00</b>  | <b>0.01</b>   | <b>0.01</b>  | –     |  |
| Exmp.trv2 | 14   | 22    | 4  | $= 2$                                | 0.00         | 0.01         | 0.01         | 0.02         | –            | $= 2$                                | 0.00         | <b>0.00</b>  | <b>0.00</b>  | <b>0.01</b>   | –            | 81.97 |  |
| Escp.4x4  | 224  | 738   | 16 | $= 2$                                | 0.08         | 11.66        | 11.73        | 1059.23      | –            | $= 2$                                | 0.08         | <b>0.13</b>  | <b>0.22</b>  | <b>0.25</b>   | –            | 99.02 |  |
| Escp.4x5  | 360  | 1326  | 20 | $= 2$                                | 0.18         | 34.09        | 33.80        | 3084.58      | –            | $= 2$                                | 0.18         | <b>0.27</b>  | <b>0.46</b>  | <b>0.63</b>   | –            | 99.02 |  |
| Escp.5x5  | 598  | 2301  | 26 | $\geq 2$                             | 0.46         | 96.24        | 97.10        | ?            | ?            | $= 2$                                | 0.46         | <b>0.68</b>  | <b>1.16</b>  | <b>1.71</b>   | –            | 98.98 |  |
| Escp.5x6  | 840  | 3516  | 30 | $\geq 2$                             | 1.01         | 217.63       | 216.83       | ?            | ?            | $= 2$                                | <b>1.00</b>  | <b>1.42</b>  | <b>2.40</b>  | <b>4.30</b>   | –            | 99.00 |  |
| Escp.6x6  | 1224 | 5424  | 36 | $\geq 2$                             | 2.13         | 516.92       | 511.41       | ?            | ?            | $= 2$                                | <b>2.06</b>  | <b>2.90</b>  | <b>5.12</b>  | <b>10.30</b>  | –            | 98.97 |  |
| Escp.7x7  | 2350 | 11097 | 50 | $\geq 2$                             | 7.81         | 2167.86      | 2183.01      | ?            | ?            | $= 2$                                | <b>7.71</b>  | <b>10.67</b> | <b>19.04</b> | <b>52.47</b>  | –            | 98.99 |  |
| Escp.7x8  | 3024 | 14820 | 56 | $\geq 0$                             | <b>13.07</b> | ?            | ?            | ?            | ?            | $= 2$                                | 13.44        | <b>18.25</b> | <b>32.69</b> | <b>108.60</b> | –            | 99.01 |  |

| Benchmark    |                 | Reduction + Yosys + SafetySynth (symbolic) |              |              |              |              |              |              | Incremental Synthesis (explicit-state implementation) |              |              |              |              |              |              |       |
|--------------|-----------------|--|--------------|--------------|--------------|--------------|--------------|--------------|---|--------------|--------------|--------------|--------------|--------------|--------------|-------|
| name         | $\delta_{\max}$ | $\delta = 0$                               | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta = 5$ | $\delta = 6$ | $\delta = 0$  | $\delta = 1$ | $\delta = 2$ | $\delta = 3$ | $\delta = 4$ | $\delta = 5$ | $\delta = 6$ | %     |
| Stub.4x4 = 2 |                 | 1.07                                       | 1.24         | 1.24         | 1.80         | –            | –            | –            | <b>0.04</b>   | <b>0.07</b>  | <b>0.12</b>  | <b>0.18</b>  | –            | –            | –            | 98.98 |
| Stub.4x5 = 2 |                 | 1.16                                       | 1.49         | 1.49         | 2.83         | –            | –            | –            | <b>0.08</b>   | <b>0.14</b>  | <b>0.25</b>  | <b>0.44</b>  | –            | –            | –            | 98.97 |
| Stub.5x5 = 2 |                 | 1.19                                       | 2.61         | 2.50         | 13.67        | –            | –            | –            | <b>0.21</b>   | <b>0.37</b>  | <b>0.63</b>  | <b>1.17</b>  | –            | –            | –            | 98.97 |
| Stub.5x6 = 2 |                 | 1.18                                       | 2.60         | 2.59         | 23.30        | –            | –            | –            | <b>0.42</b>   | <b>0.69</b>  | <b>1.20</b>  | <b>2.49</b>  | –            | –            | –            | 98.96 |
| Stub.6x6 = 4 |                 | 1.17                                       | 2.76         | 2.74         | 19.96        | 19.69        | 655.24       | –            | <b>0.93</b>   | <b>1.47</b>  | <b>2.60</b>  | <b>5.79</b>  | <b>7.54</b>  | <b>7.60</b>  | –            | 99.89 |
| Stub.7x7 = 4 |                 | <b>1.23</b>                                | <b>2.50</b>  | <b>2.48</b>  | 24.57        | <b>23.01</b> | 2224.62      | –            | 3.60  | 5.52         | 10.08        | <b>22.75</b> | 31.18        | <b>32.98</b> | –            | 99.88 |

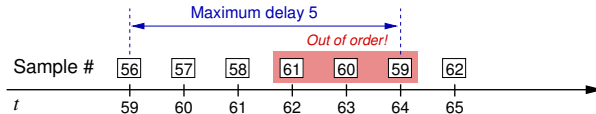
# Out-of-Order Message Delivery

☹ Observations may arrive *out-of-order* :

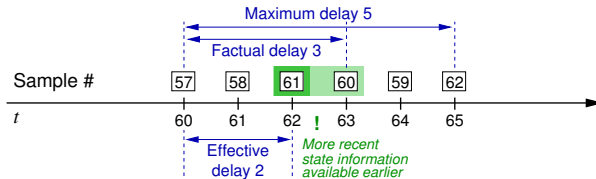


# Out-of-Order Message Delivery

- ☹ Observations may arrive *out-of-order* :

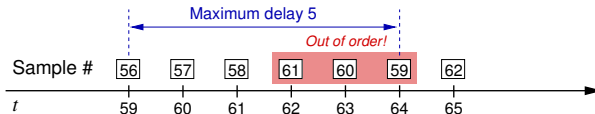


- 😊 But this may only reduce effective delay, improving controllability :

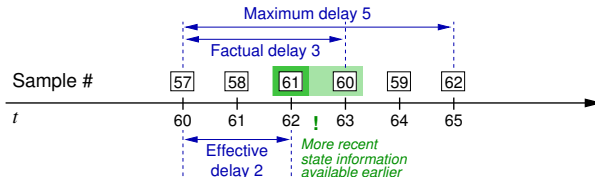


# Out-of-Order Message Delivery

- ☹ Observations may arrive *out-of-order* :



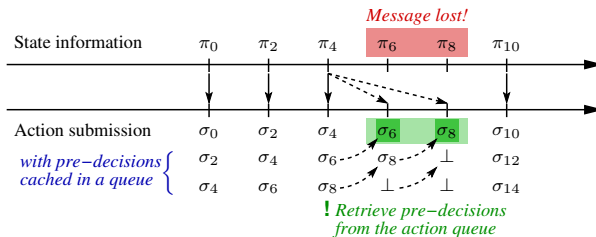
- 😊 But this may only reduce effective delay, improving controllability :



- 😊 W.r.t. qualitative controllability, the **worst-case of out-of-order** delivery is **equivalent to order-preserving delay  $k$** .
- 😊 Stochastically **expected controllability even better** than for strict delay  $k$ .

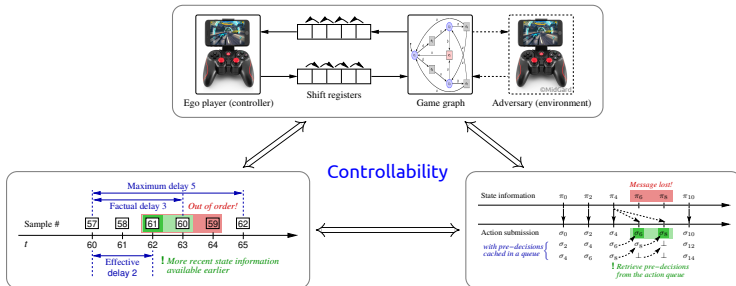
# (Bounded) Message Loss

- ☹ Message carrying the state information may get *lost* :



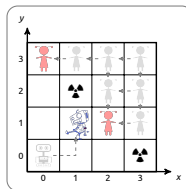
- 😊 The controller can **still win** a safety game in the presence of bounded message loss **leveraging delay-resilient strategies**.

# Equivalence of Qualitative Controllability

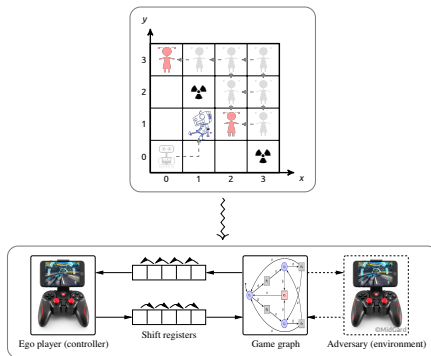


⇒ M. Chen, M. Fränzle, Y. Li, P. N. Mosaad, N. Zhan : *Indecision and delays are the parents of failure : Taming them algorithmically by synthesizing delay-resilient control*. Acta Informatica '20.

# Summary

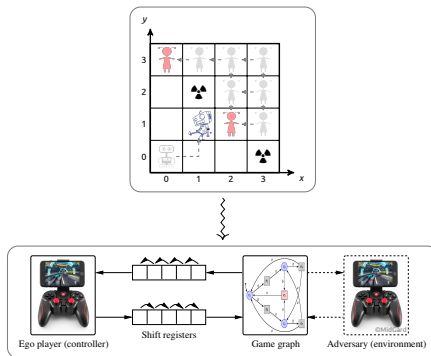


# Summary



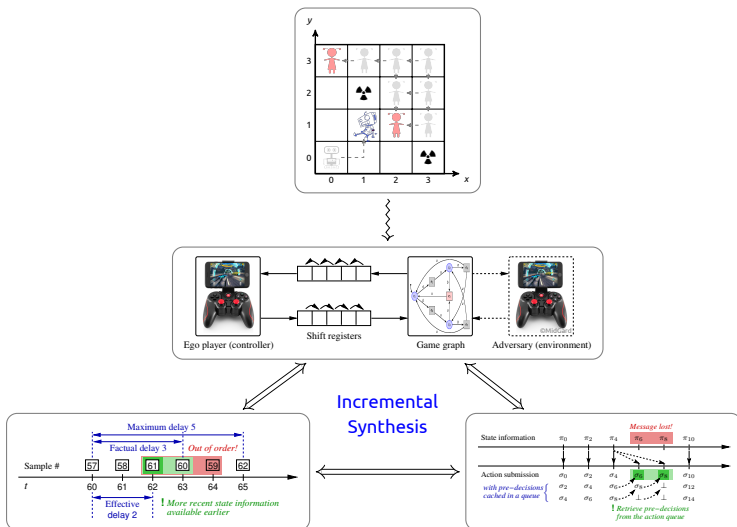


# Summary

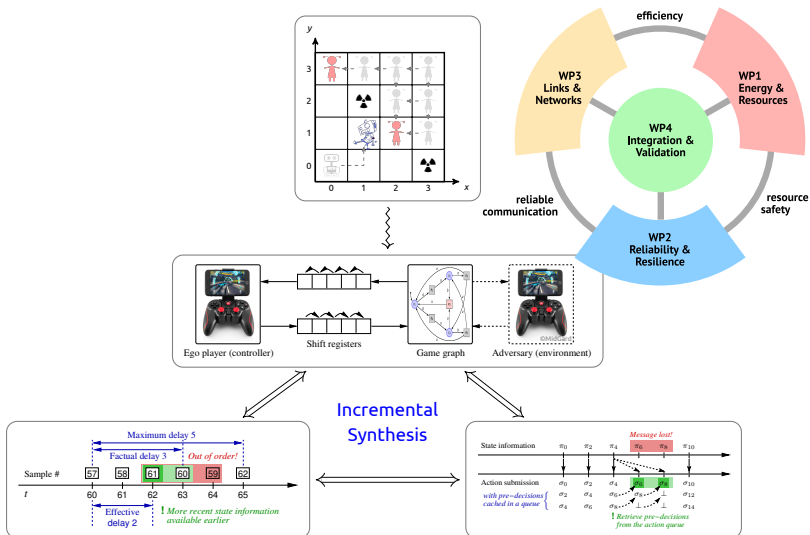


## Incremental Synthesis

# Summary



# Summary

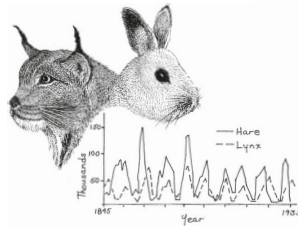


# Delays in Dynamical/Hybrid Systems



©Wikipedia

Vito Volterra



©J. Pastor, 2016

Predator-prey dynamics

*"[...] the rate of change of physical systems depends not only on their present state, but also on their past history."*

[Richard Bellman and Kenneth L. Cooke, 1963]

- ⇒ M. Chen, M. Fränzle, Y. Li, P. N. Mosaad, N. Zhan : *Validat. simul.-based verif. of DDEs*. FM '16.
- ⇒ B. Xue, P. N. Mosaad, M. Fränzle, M. Chen, Y. Li, N. Zhan : *Safe approx. of reach. sets for DDEs*. FORMATS '17.
- ⇒ S. Feng, M. Chen, N. Zhan, M. Fränzle, B. Xue : *Taming delays in dyn. syst. : Unbounded verif. of DDEs*. CAV '19.

# HURRAY FOR DELAY!

Brussels Dichterscollectief  
Le Collectif de Poètes Bruxellois  
Brussels Poetry Collective

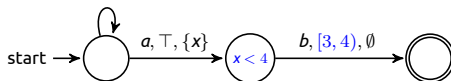
|      |       |
|------|-------|
| 2011 | 27.03 |
|------|-------|

© Brussels Poetry Collective

## Q1a : Resultant Forms of Delay

**Delayed reaction** : Reaction to a stimulus is not immediate.

- Easy to model in timed/hybrid automata, etc. :



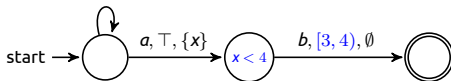
- Thus amenable to the pertinent analysis tools.

⇒ **Not of interest today.**

## Q1a : Resultant Forms of Delay

**Delayed reaction** : Reaction to a stimulus is not immediate.

- Easy to model in timed/hybrid automata, etc. :



- Thus amenable to the pertinent analysis tools.

⇒ **Not of interest today.**

**Network delay** : Information of different age coexists and is queuing in the network when piped towards target.

- End-to-end latency may exceed sampling intervals etc. by orders of magnitude.
- Not (efficiently) expressible in standard models.

⇒ **Our theme today : discrete-time pipelined delay.**

[Chen *et al.* : ATVA '18, Acta Inf. '20];

[Zimmermann : LICS '18, GandALF '17], [Klein & Zimmermann : ICALP '15, CSL '15].

# Equivalence of Qualitative Controllability

## Theorem (Equivalence of qualitative controllability)

Given a two-player safety game, the following statements are equivalent if  $\delta$  is even :

- 1 *There exists a winning strategy under an exact delay of  $\delta$ , i.e., if at any point of time  $t$  the control strategy is computed based on a prefix of the game that has length  $t - \delta$ .*
- 2 *There exists a winning strategy under time-stamped out-of-order delivery with a maximum delay of  $\delta$ , i.e., if at any point of time  $t$  the control strategy is computed based on the complete prefix of the game of length  $t - \delta$  plus potentially available partial knowledge of the game states between  $t - \delta$  and  $t$ .*
- 3 *There exists a winning strategy when at any time  $t = 2n$ , i.e., any player-0 move, information on the game state at some time  $t' \in \{t - 2k, \dots, t\}$  is available, i.e., under out-of-order delivery of messages with a maximum delay of  $\delta$  and a maximum number of consecutively lost upstream or downstream messages of  $\delta/2$ .*

The first two equivalences do also hold for odd  $\delta$ .

⇒ M. Chen, M. Fränzle, Y. Li, P. N. Mosaad, N. Zhan : *Indecision and delays are the parents of failure : Taming them algorithmically by synthesizing delay-resilient control*. Acta Informatica '20.